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B.Sc Part III

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Ring of Polynomials

~~Let f(x)~~

Th The set $R(x)$ of all polynomials over an arbitary ring R is a ring w.r.t addition and Multiplication of Polynomials.

Proof Let $f(x), g(x) \in R[x]$. Then $f(x)+g($ and $f(x) \cdot g(x)$ are also polynomials over R. Therefore $R(x)$ is closed w.r.t addition and Multiplication of Polynomials.

Now let

$$f(x) = \sum a_i x^i = a_0 x^0 + a_1 x^1 + a_2 x^2 + \text{---} + \cdots$$

$$g(x) = b_0 x^n + b_1 x + b_2 x^2 + \text{---}$$

$$h(x) = c_0 x^n + c_1 x + c_2 x^2 + \text{---}$$

be any arbitary elements of $F(x)$

Commutative law: We have

$$f(x)+g(x) = (a_0+b_0)x^0 + (a_1+b_1)x + (a_2+b_2)x^2 + \text{---}$$

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$= (b_0 + a_0)x^0 + (b_1 + a_1)x + (b_2 + a_2)x^2$

$+ \text{---} = g(x) + f(x)$

II **Associativity of addition** We have

$(f(x) + g(x)) + h(x) = \sum (a_i + b_i)x^i + \sum c_i x^i$

$= \sum [(a_i + b_i) + c_i] x^i$

$= \sum (a_i + (b_i + c_i$

$\therefore$ the zero polynomial ~~identity~~

$O(x)$ is the additive identity.

Existence of additive Inverse. Let $-f(x)$ be the polynomial over R defined as $-f(x) = (-a_0)x^0 + (-a_1) + (-a_2)x^2 + \_\_\_\_$

Then $-f(x) + f(x) = (-a_0 + a_0)x^0 + (-a_1 + a_1)x + (-a_2 + a_2)x^2 + \_\_\_\_$
$= 0x^0 + 0x + a x^2 + \_\_\_\_$ ~~= the d~~
$=$ the additive identity.
$\therefore$ Each member of $R(x)$ possesses additive inverse.

Associativity of Multiplication

We have
$f(x)\, g(x) = (a_0 x^0 + a_1 x + a_2 x^2 + \_\_\_\_)$
$(b_0 x^0 + b_1 x + b_2 x^2 + \_\_\_\_)$
$= d_0 x^0 + d_1 x + d_2 x + \_\_ + d_l x^l$

$+ \_\_\_$ where $d_l = \sum_{i+j=l} a_i b_j$

Now $[f(x)\, g(x)]\, h(x)$

~~Where~~

Now $[f(x)\, g(x)]\, h(x)$

$$= (d_0 x^0 + d_1 x + d_2 x^2 + \ldots)(c_0 x^0 + c_1 x + c_2 x^2 + \ldots)$$

$$= e_0 x^0 + e_1 x + e_2 x^2 + \ldots + e_n x^n$$

where e_n = the coefficient of x^n in $[f(x) \cdot g(x)]\, h(x)$

$$= \sum_{l+K=n} d_l c_K = \sum_{l+K=n} \left[\left(\sum a_i b_j\right) c_K\right]$$

$$= \sum_{i+j+K=r} a_i b_j c_K$$

Similarly we can show that the coefficient of x^n in

$$f(x) \cdot [g(x) \cdot h(x)] = \sum_{i+j+K} a_i b_j c_K$$

Thus $[f(x) \cdot g(x)]\, h(x) = f(x)[g(x) \cdot h(x)]$

Since corresponding coefficients in these two polynomials are equal.

Similarly distributivity of Multiplication with respect to addition can be hold. (try by yourself)